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Analysis of low Reynolds number flow around a heated circular cylinder †

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Abstract

The objective of this study is to investigate the forced convection from and the flow around a heated cylinder. Experimental and computational results are presented for laminar flow around a heated circular cylinder with a diameter of 10 mm. The experiments were carried out using Particle Image Velocimetry (PIV) in a wind tunnel, and numerical simulations using an in-house code and a commercial software package, FLUENT. This paper presents comparisons for vorticity and temperature contours in the wake of the cylinder. Experimental and computational results are compared with those available in the literature for heated and unheated cylinders. An equation is suggested for a temperature-dependent coefficient defining a reference temperature to be used in place of the constant used in other studies. An attempt is also made to correct differences between average cylinder surface temperature and measured interior temperature of the cylinder.

Keywords: CFD; Circular cylinder; Heated cylinder; Nusselt number; PIV; Reynolds number

1. Introduction

The flow around cylindrical bodies has been the subject of intense research for decades, and heat transfer is also of key importance. Here we investigate the effect of heat transfer on the flow around a heated cylinder. Our eventual aim is to design a measuring method for the real-time monitoring of the velocity profile in a channel with a large cross-section [1, 2].

Since the properties of the fluid depend on temperature, and for a heated cylinder the temperature of the fluid varies, it is necessary to consider the most appropriate temperature for calculating the fluid properties and related similarity numbers [2]. A typical approach is to use the average of the wall and ambient temperatures [3, 4].

Here we present experimental and computational results for laminar flow around a heated circular cylinder and compare computed vorticity and temperature contours in the wake of the cylinder. We also discuss which temperature should be used to represent the air properties in the similarity numbers.

2. Measuring setup and results

Experiments were carried out at the Laboratory of Fluid Dynamics and Technical Flows, University of Magdeburg, in a Göttingen type wind tunnel with an open test section (height 500 mm, width 600 mm, length 1070 mm). A D=10 mm diameter and L=600 mm length circular cylinder was placed horizontally with its axis perpendicular to the free stream velocity U (see layout in Fig. 1). The cylinder was heated electrically, kept at a constant temperature T_w with a potentiometer, and its temperature was measured by a thermocouple.

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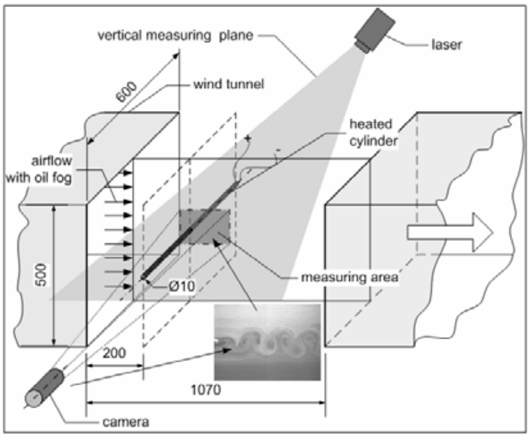

Fig. 1. Schematic of experimental setup.

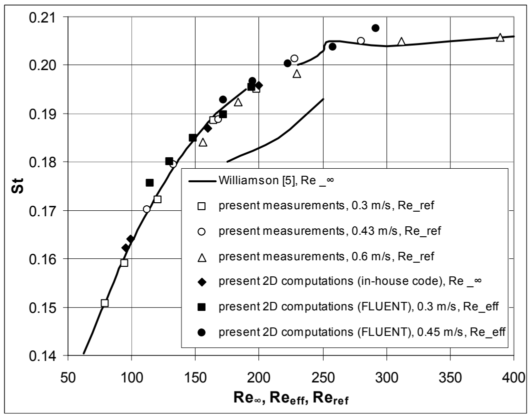

Fig. 2. Strouhal number versus Reynolds number for experimental and computational results, compared with Williamson [5].

The flow around the cylinder was measured in a measuring area consisting of a vertical plane perpendicular to the axis of the cylinder. Measurements were carried out using PIV, allowing us to measure a two-dimensional cross-section of the flow field. Oil fog was added to the flow and the measurement area was illuminated by a laser sheet. The laser acts as a photographic flash for the digital camera, and the particles in the fluid scatter the light. Using this method, we are able to obtain the two-dimensional velocity distribution at different instances. From the velocity distribution streamlines can be determined.

From the velocity field **v** we can obtain the vorticity distribution $\varsigma = curl\, \mathbf{v}$ by numerical differentiation. The vorticity distribution shows the precise location, centre, and direction of rotation, allowing us to identify the structure of flow.

Measurements were carried out at the three velocities of U=0.3, 0.43, 0.6 m/s, corresponding to $\mathrm{Re}_\infty = U\,D/\nu_\infty = 195, 277, 388$, and at five cylinder temperature values (T_w = 297, 373, 473, 573, 673 K). Here ν_∞ is the kinematic viscosity of the ambient air (at T_∞). The ambient temperature, and also the temperature of the unheated cylinder, was T_w=T_∞=297K. For the heated cylinder, when calculating similarity numbers, Özisik [3] and Bejan [4] suggest basing them on the average or film temperature of the cylinder and the ambient air

$$T_f = \frac{1}{2}\left(T_w + T_\infty\right). \tag{1}$$

Here we define the reference temperature as

$$T_{ref} = T_\infty + c_{ref}\left(T_w - T_\infty\right), \tag{2}$$

which is reduced to Eq. (1) when $c_{ref} = c_f = 0.5$, where subscript f refers to the film value.

The kinematic viscosity belonging to the reference temperature $\nu_{ref} = \nu\left(T_{ref}\right)$ and the related Reynolds number $\mathrm{Re}_{ref} = U\,D/\nu_{ref}$ – which takes into account also the cylinder temperature – can also be calculated. The dimensionless vortex shedding frequency or Strouhal number St $= fD/U$ for an unheated cylinder is available as a function of Reynolds number [5, 6].

One set of data consisting of 30 photographs is available for each combination of velocity and temperature. The time-history of the velocity was measured using LDA at a point 70 mm downstream of the center of the cylinder and then applied FFT analysis to the signal to determine the vortex shedding frequency.

The results obtained from an analysis of the results can be found in Fig. 2, along with computational results and those of Williamson [5], whose data are for an unheated cylinder at varying free stream velocity U.

The current measurements, however, refer to three constant velocities with the cylinder temperatures varying. We found good agreement with Williamson's results over the domain $\mathrm{Re}_{ref} = 79 - 390$ if c_{ref} is defined as a function of temperature ratio $T^* = T_w / T_\infty$ as

$$c_{ref} = 0.135\left(T^*\right)^3 - 0.832\left(T^*\right)^2 + 1.626\,T^* - 0.432 \tag{3}$$

shown in Fig. 3.

Wang et al. [7], based on their experiments with a cylinder of 1.07 mm in diameter, offer a value of c=0.28 independently of the temperature ratio in the domain of $T^* < 2$ and call the related temperature

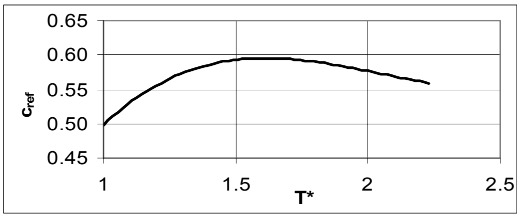

Fig. 3. c_{ref} as a function of temperature ratio.

$T_{eff} = T_\infty + 0.28(T_w - T_\infty)$ effective temperature. The question arises whether the substantial difference between our results and those of Wang et al. is caused by the substantial difference in cylinder diameters. It is worth mentioning, however, that if we replace the 0.28 value of the c suggested by Wang et al. [7] by the c_{ref} defined by equation (3), our test results for Strouhal number differ less than 1% from their values, approximated by $\mathrm{St} = 0.2660 - 1.0160/\sqrt{\mathrm{Re}_{eff}}$ for $\mathrm{Re}_{eff} < 300$.

One objective of this paper is to determine the Nusselt number. First we checked the criteria for forced convection based on formulae in Wang and Trávniček [2], and found that all formulae predict forced convection for all of our experimental tests. The heat loss of the cylinder is due to heat convection and radiation, and we neglected the heat conduction, based on Wang and Trávniček's findings that heat conduction is negligible. We measured the current $I\left[A\right]$ and voltage $U_e\left[V\right]$ of the direct current used for heating of the cylinder. We assumed that the electrical power $P_e = U_e I$ is fully converted into heat. The cylinder was painted matt black so its emissivity is $\varepsilon \approx 0.97$. The volumetric coefficient of thermal expansion of air is $\beta = 1/(273.15\ K)$, and its heat conduction coefficient k changes with absolute temperature T as $k = 0.0703\,T + 5.5306$. The Stefan-Boltzmann constant is $\sigma = 5.6705\times10^{-8}\ W\,m^{-2}\,K^{-4}$. Based on the results of Wang and Trávniček [2] the Nusselt number belonging to film temperature Nu_f can be written as follows:

$$\mathrm{Nu}_f = \frac{1}{\pi\,k_f\left(T_w - T_\infty\right)}\left[\frac{P_e}{L} - \pi D\,\varepsilon\,\sigma\left(T_w^4 - T_\infty^4\right)\right]. \quad (4)$$

Wang and Trávniček [2] showed that this Nusselt number is a linear function of the square root of the Reynolds number Re_{rep} based on the representative temperature defined by

$$T_{rep} = T_\infty + 0.36\left(T_w - T_\infty\right)$$

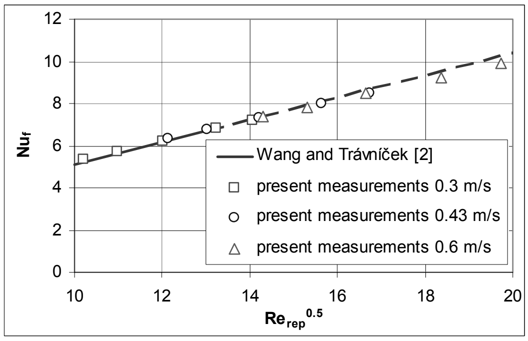

Fig. 4. Nusselt number versus square-root of Reynolds number Re_{rep} for experimental results, compared with Wang and Trávniček [2].

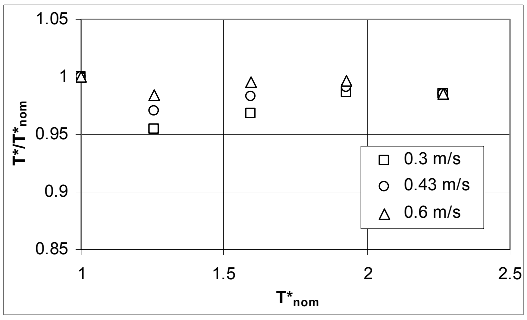

Fig. 5. Correction of surface temperature.

over the domain of $40 \le \mathrm{Re}_{rep} \le 150$, i.e.,

$$\mathrm{Nu}_f = -0.153 + 0.527\ \mathrm{Re}_{rep}^{0.5}. \quad (5)$$

We extended the domain of investigation to $\mathrm{Re}_{rep} \le 340$. Fig. 4 shows our test results together with those of Wang and Trávniček [2] defined by Eq. (5). To make our results collapse on the curve, we had to introduce a nominal wall temperature $T_{w,nom}$ and temperature ratio $T^*_{nom} = T_{w,\,nom}/T_\infty$, respectively, and to introduce the correction between temperature ratios T^*_{nom} and $T^* = T_w/T_\infty$ shown in Fig. 5. The reason for this is that the average surface temperature T_w of the heated cylinder at this cylinder diameter is lower than the nominal temperature $T_{w,\,nom}$ measured in the interior of the cylinder. This case differs from very small diameter cylinders (e.g., [7]), where the interior and surface temperatures can hardly be distinguished. The question arises how the average cylinder surface temperature should be measured; this is a future task for investigation.

It is worth mentioning that if we calculate the average Nusselt number from the formula

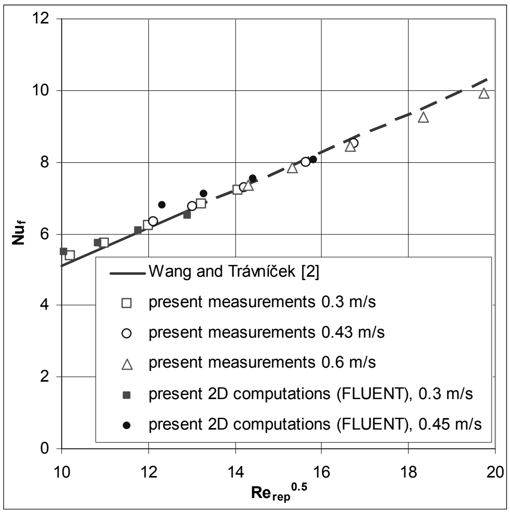

Fig. 6. Nusselt number versus Reynolds number for our experimental results evaluated by Hilpert's [8] formula, and computational results (FLUENT) compared with those of Wang and Trávniček [2].

$$\overline{\mathrm{Nu}} = 0.615\left[\mathrm{Re}_{eff}\,(T_w/T_\infty)^{0.25}\right]^{0.466}$$

suggested by Hilpert [8] using Reynolds number Re_f based on the film temperature (1)

$$\mathrm{Nu}_f = 0.615\left[\mathrm{Re}_f\,(T_w/T_\infty)^{0.25}\right]^{0.466}$$

then our results approximately collapse on the curve suggested by Wang and Trávniček [2], as can be seen in Fig. 6. Computational results can also be seen in Fig. 2: filled diamonds show Strouhal numbers obtained by using the in-house code (see section 3) and the filled squares and circles those obtained by FLUENT (see section 4).

3. Two-dimensional computations with an in-house code

In addition to experiments, numerical simulations were also performed using two different numerical tools. One of them is a 2D in-house code based on a finite difference solution of the governing equations investing low Reynolds number constant property incompressible Newtonian fluid flow around a stationary cylinder. The governing equations are the unsteady Navier-Stokes equations, continuity, a Poisson equation for pressure and an energy equation, all in dimensionless form. The computational domain is characterized by two concentric circles: the inner represents the

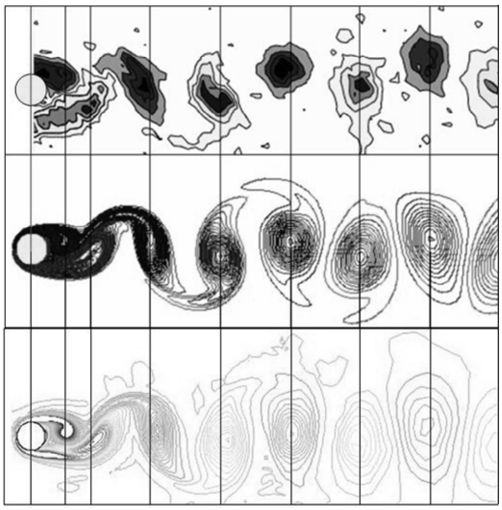

Fig. 7. Vorticity contours obtained experimentally (top) and numerically (middle: in-house code, bottom: FLUENT) for unheated cylinder at Re_∞=200.

cylinder surface, the outer the far field. Typical boundary conditions are used for velocity and pressure. The cylinder surface is kept at constant temperature T_w and the ambient temperature T_∞ is also considered to be constant. Forced convection is assumed and thus the buoyancy term is neglected in the Navier-Stokes equations. In this way the flow field is not affected by the temperature but naturally the velocity distribution influences the temperature field. With this method very good agreement was found with the Nusselt number against Reynolds number obtained experimentally (see Baranyi [9]). The computational method and its validation are described in detail in Baranyi [9, 10].

Computations were carried out for both heated and unheated stationary cylinder. The obtained Strouhal numbers can be seen as filled diamonds in Fig. 2. Figure 7 shows vorticity contours belonging to the same instant at Re_∞=200 obtained from experimental data (top), computational results by in-house code (middle) and FLUENT (bottom). The computational results compare well with experimental and with other computational results.

4. Numerical simulation with commercial software

For the numerical solution of the governing equations in two-dimensions the commercial software package FLUENT V6.3.26 is employed, which uses the finite volume method. The top and bottom boundaries are modeled as symmetric, the inlet velocity are U=0.3 and 0.45 m/s, and ambient temperature is constant (T_∞=297 K).The effect of gravity is neglected. The cylinder surface is kept at different constant temperatures of T_w=297, 373, 473, 573, 673 K. The computational domain is characterized by two concentric circles: the inner represents the cylinder surface, the outer the far field. The numerical grid consists of 28800 elements. The fluid properties are not constant; the effect of temperature is taken into account.

As mentioned earlier, computational results for Strouhal number are shown and compare reasonably well with the results of Williamson [5] for unheated cylinders, if Re is computed based on effective temperature as suggested by Wang et al. [7], shown in Fig. 2. Fig. 6 shows the calculated film Nusselt number Nu_f using Hilpert formula versus the square-root of Re_{rep}. Results compare reasonably well with results on the straight line suggested by Wang and Trávniček [2]. Finally, the bottom figure in Fig. 7 shows vorticity contours obtained by FLUENT, and again compare well with experimental and other computational data.

5. Conclusions

Our experimental and computational results for heated and unheated cylinders in terms of St-Re relationship agree reasonably well (by introducing effective and representative temperatures) with those of Williamson [5] obtained for unheated cylinder.

As a result of our investigations, a new c_{ref} coefficient was derived for the reference temperature T_{ref} defined in Eq. (2) which is a function of the temperature ratio (see Eq. (3) and Fig. 3), instead of the constant given in the literature (e.g., [7]).

In this study the domain of validity for the relationship $\mathrm{Nu}_f\left(\mathrm{Re}_{ref}\right)$ available in the literature was extended from Re=150 [2] up to Re=340 (see Fig. 4). To facilitate the extension of the Reynolds number domain, the relationship between average cylinder surface temperature and average interior temperature was determined (see Fig. 5).

Further investigation is required to determine the relationship between average surface temperatures and average interior temperatures at different cylinder diameters.

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References

[1] Sz. Szabó and A. Juhász, Messung der Geschwindigkeitsverteilung in großen Strömungs-querschnitten, *VGB PowerTech*, 7 (2003) 51-56.

[2] A.-B. Wang and Z. Trávniček, On the linear heat transfer correlation of heated circular cylinder in laminar crossflow using new representative temperature concept, *International Journal of Heat and Mass Transfer*, 44 (2001) 4635-4647.

[3] M. N. Özisik, *Heat Transfer*, McGraw Hill, New York, (1985).

[4] A. Bejan, *Heat Transfer*, John Wiley & Sons, New York, (1993).

[5] C. H. K. Williamson, Vortex dynamics in the cylinder wake, *Annual Review of Fluid Mechanics*, 28 (1996) 477-539.

[6] C. Norberg, Flow around a circular cylinder: Aspects of fluctuating lift, *Journal of Fluids and Structures*, 15 (2001) 459-469.

[7] A.-B. Wang, Z. Trávniček and K.C. Chia, On the relationship of effective Reynolds number and Strouhal number for the laminar vortex shedding of a heated circular cylinder, *Physics of Fluids*, 12 (6) (2000) 1401-1410.

[8] R. Hilpert, Wärmeabgabe von geheizten Drähten und Rohren im Luftstrom, *Forschung Gebiete Ingenieur Wesens*, 4 (1993) 215-224.

[9] L. Baranyi, Computation of unsteady momentum and heat transfer from a fixed circular cylinder in laminar flow, *Journal of Computational and Applied Mechanics*, 4 (2003) 13-25.

[10] L. Baranyi, Numerical simulation of flow around an orbiting cylinder at different ellipticity values, *Journal of Fluids and Structures*, 24 (2008) 883-906.

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